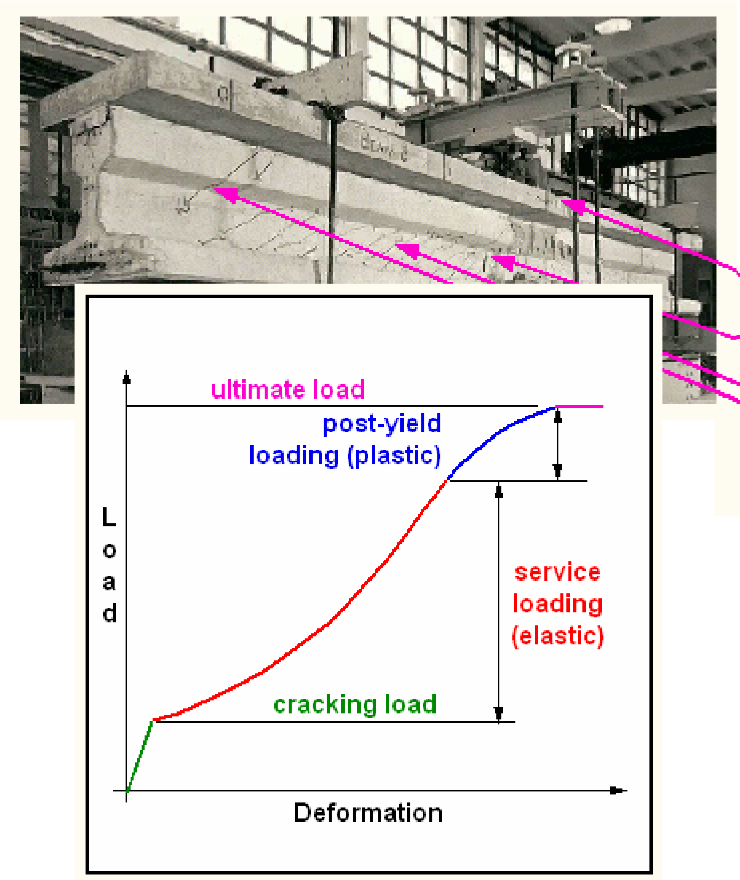
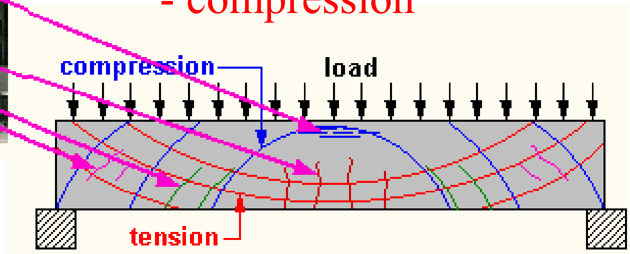
Behavior of a reinforced concrete beam

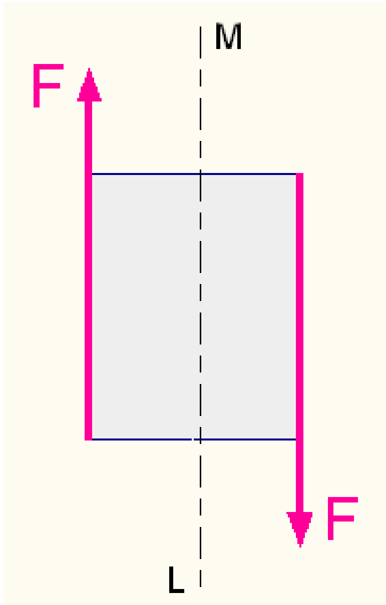


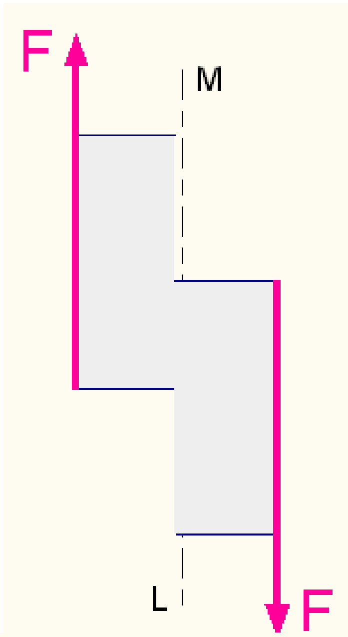
Cracks
- tensile vertical
- tensile inclined
- shear
- compression

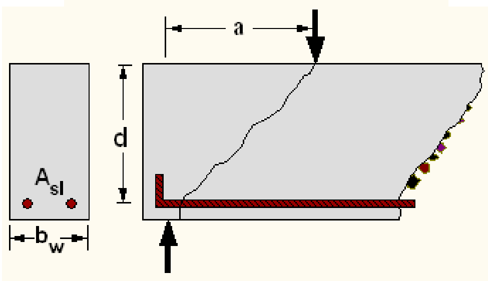


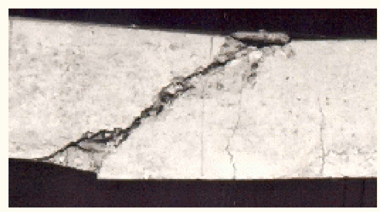
The beam may have several possible modes of failure:
- cracking
- deflection
- shear
- bending

What is shear of a beam

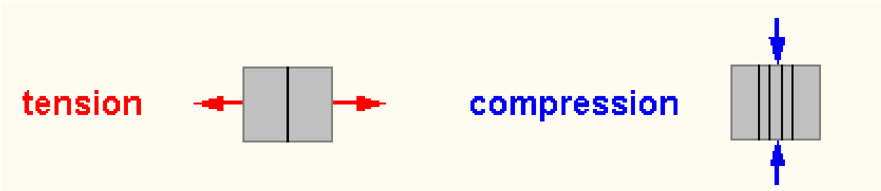


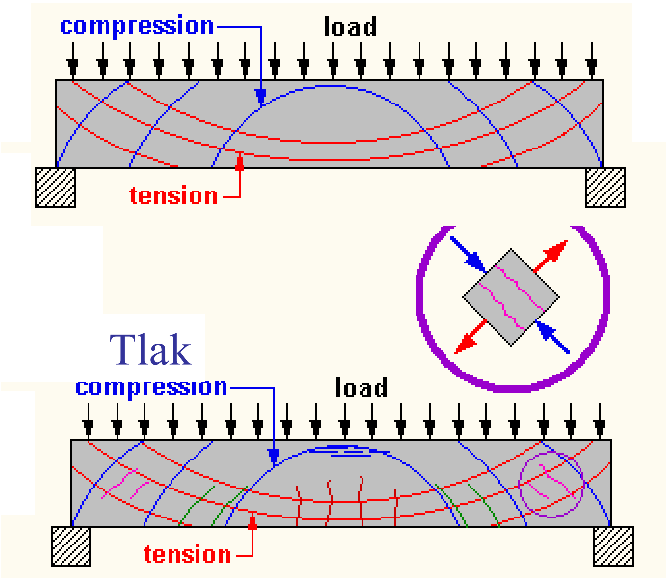




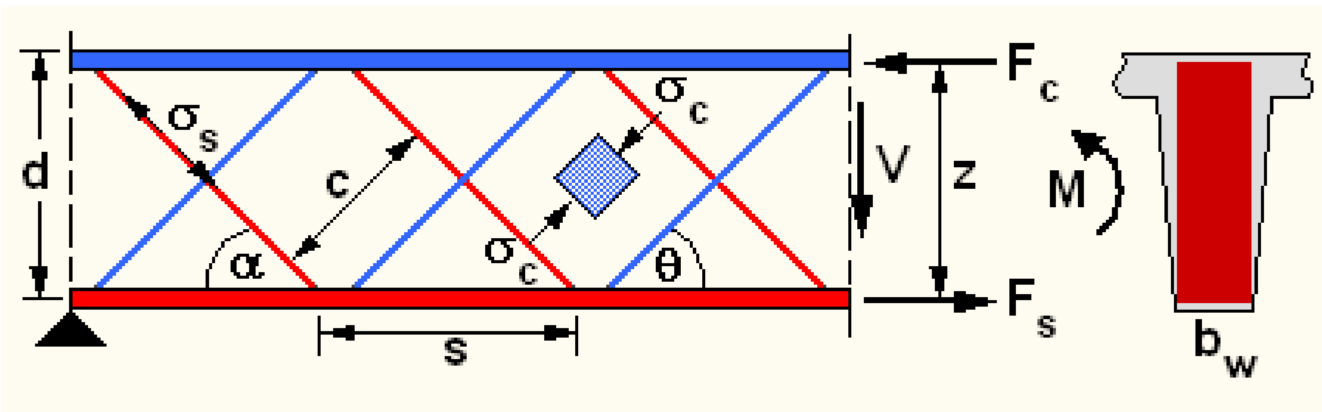


The principles stresses - crack formation





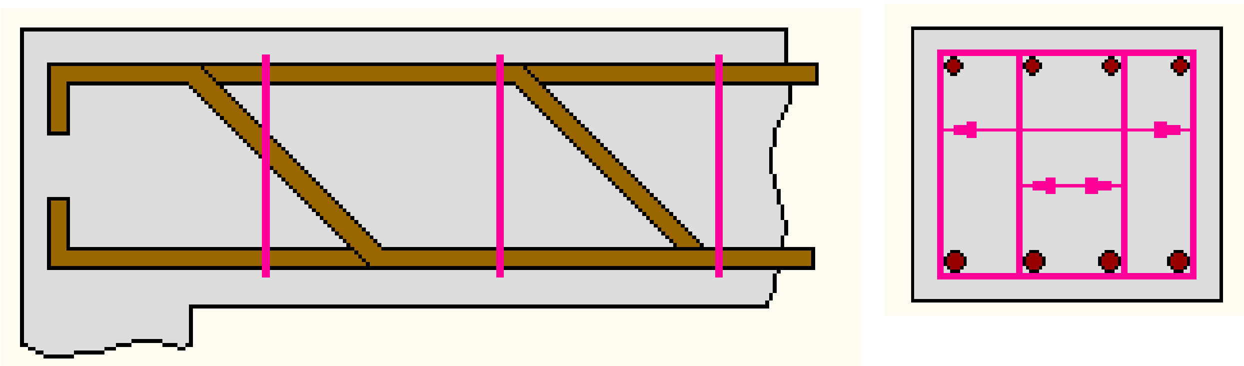
Truss analogy



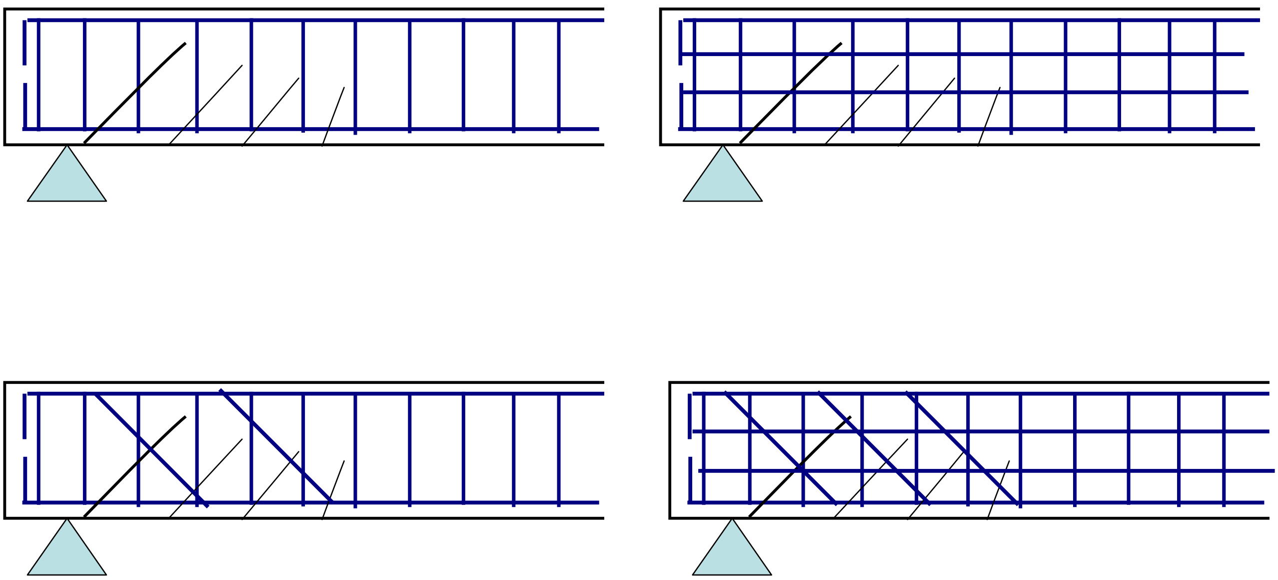
Blue: compression elements

Red: tension elements

Vertical links and bent up bars

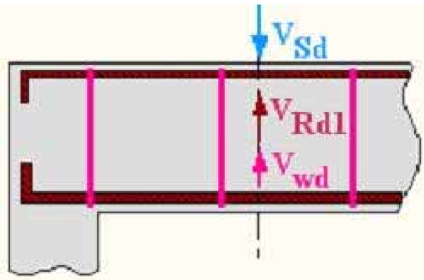


Shear reinforcement of a beam



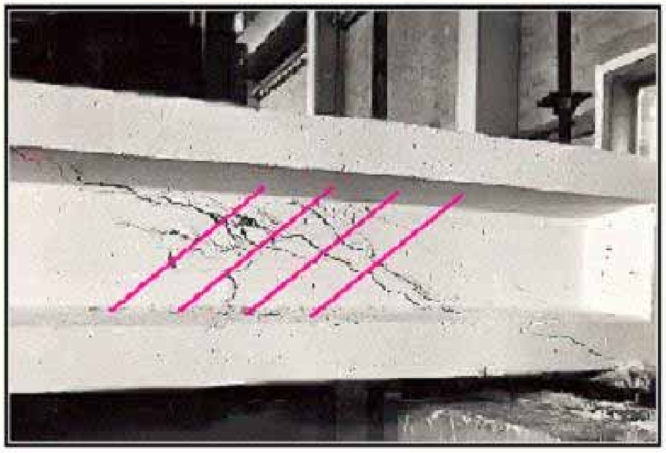
Shear - Basic requirement

$$V_{Rd1} + V_{wd} \geq V_{Sd}$$



V_{Sd1} load effect shear foorce
V_{Rd1}combine shear resistanec of the concrete and longitudinal bars,
V_{wd} shear resistance due to additional shear reinforcenment

Inclined bars are more difficult to fix



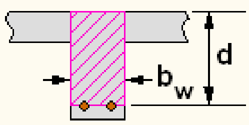
Shear capacity of a beam

It is generally regarded that there are 3 factors which contribute to the resistance of a reinforced concrete member which has NO shear reinforcement :-

1) the area of concrete in compression
2) the dowel action of the longitudinal tensile reinforcement
3) the interlocking nature of the aggregate

A fully developed plastic theory exists, backed-up by experiments, to determine the shear behaviour of reinforced concrete beams without shear reinforcement. However, the theory is very complicated, and it was decided to adopt a traditional, empirical approach in the Code.

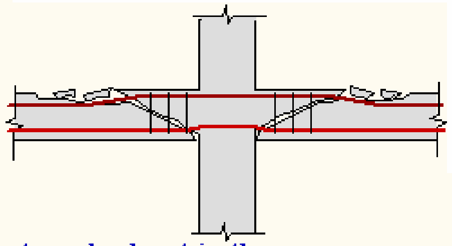
This assumes that the shear resistance, V_{Rd1} (or V_{cd}) is derived from a shear capacity (stress), τ_c acting uniformly over the effective area of the section, thus:



$$V_{Rd1} = \tau_c \, b_w \, d$$

Extensive tests have shown that τ_c is influenced by the parameters shown on the buttons above.

Dowel effect



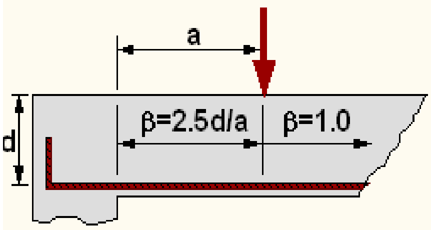
Once punching has occurred, the top bars make only a very limited contribution to the shear resistance since the cover is easily torn away (but prior to punching they are vital to the truss analogy in determining the strength).

However, the bottom bars, being more deeply embedded, are not pushed out in the same way and thus provide more resistance. This is attributable initially to dowel action, and then later at larger deformations to its being kinked, as shown.

When punching occurs at a slab-column connection without shear reinforcement the resistance and thus the load carrying capacity is greatly reduced. The load is therefore transferred to adjacent connections, which may also suffer punching failures. This may lead to a general failure of the floor which in turn could lead to a progressive collapse of the structure as one floor falls onto the floor below. This has occurred several times with flat slab structures in recent years.

Providing shear reinforcement to restrain the top bars by tying them to the bottom bars, greatly increases the resistance and the ductility of the slab-column connection. Running the bottom bars through the column or anchoring them in the column will also increase the ductility.

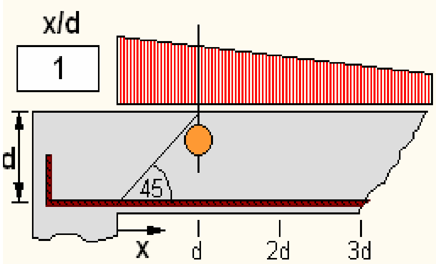
Shear capacity of a beam



The formula for shear capacity, $\tau_c = \tau_{Rd}\, k\, (1.2 + 40\, \rho_l) + \underline{|0.15|}\, \sigma_{cp}$ makes no allowance for the increased shear strength for small values of a/d at *direct supports*, where there is a clear strut or arch action.

For concentrated loads within 2.5d from the face of the support the shear capacity can be enhanced by increasing the basic shear strength, τ_{Rd} by a factor β, whose value is inversely proportional to a, thus $\beta = 2.5d/a$, but limited to $\underline{|5.0|}$. On the span side of the load β is 1.0.

The increase in shear capacity is only permissible if the tension reinforcement is *fully anchored*.



For continuous distributed loading , as indicated by the shear force diagram, the shear capacity is unchanged. Instead a reduction in shear force is generally effected by not considering the design shear force nearer than an effective depth, d from the face of a direct support.

This is based on a 45^o projection from the face of the support at the effective depth.

Concrete strenght and scale effect

Values of τ_{Rd} N/mm^2 (with $\gamma_c = 1.5$)							
f_{ck}	12	16	20	25	30	35	≥40
$f_{ctk\ 0.05}$	1.1	1.3	1.5	1.8	2.0	2.2	2.5
τ_{Rd}	0.18	0.22	0.26	0.30	0.34	0.37	0.41

k is a coefficient to allow for the 'scale effect'.

k = 1 for members where more than 50% of the longitudinal tensile reinforcement has been curtailed.

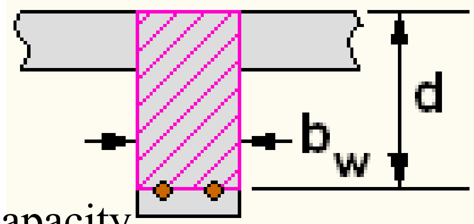
otherwise,

$k = (1.6 - d) \nless 1$ (where d is in metres)

This means that members with d < 600 mm will have an enhanced shear capacity.

An example

Concrete C20, $f_{ck} = 20$ MPa, $f_{ctk\ 0,05} = 1,5$ MPa, $\tau_{Rd} = 0,26$ MPa



$$V_{cd} = \tau_c\, b_w\, d$$

Shear capacity for $\rho = 0,01$

$$\tau_c = \tau_{Rd}\, k\,(1.2 + 40\,\rho_l) + |\,0.15\,|\,\sigma_{cp}$$

$$\tau_c = 0,26 \times 1 \times (1,2 + 40 \times 0,01) + 0 = 0,42 \text{ MPa}$$

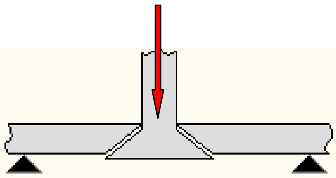
A beam $b_w = 0,30$ m, $d = 0,40$ m:

$$V_{cd} = 0,42 \times 0,30 \times 0,40 = 0,05 \text{ MN} = 50 \text{ kN}$$

Slab beam $b_w = 1,00$ m, $d = 0,17$ m:

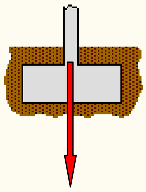
$$V_{cd} = 0,42 \times 1,00 \times 0,17 = 0,071 \text{ MN} = 71 \text{ kN}$$

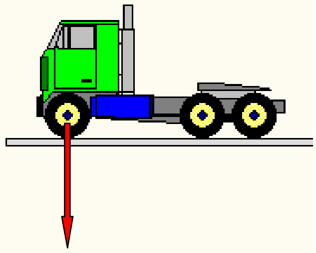
Punching shear



Consider a portion of slab subjected to an increasing concentrated load. Eventually the slab will fail. One possible method of failure is that the load 'punches' through the slab, thus:

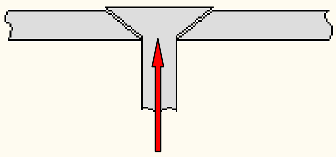
The failure mechanism is by shear, hence the name Punching Shear.





Some examples of the occurence of concentrated loads on a slab are a column, particularly on a pad foundation, and wheel loads.

This same type of failure could also happen in another way. Turning the structure upside down we get .... a flat slab supported by a column, where, as described in the previous topic 'Slabs', there is a high concentration of shear force around the column head.



When the total shear force exceeds the shear resistance of the slab, the slab will be 'pushed down' around the column, or this can be viewed as the column being 'punched' through the slab, thus:

Punching shear is most common, and is a major design consideration, in flat slab construction. In pad foundations, where weight and depth are not so critical, its effects are satisfied by providing sufficient depth. The major emphasis of this topic is, therefore, concentrated on flat slabs.

Pipers Row Car Park, Wolverhampton, 1997

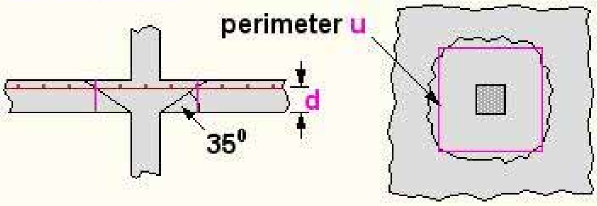
CHDG Airport - cracks of the outer surface of the shell

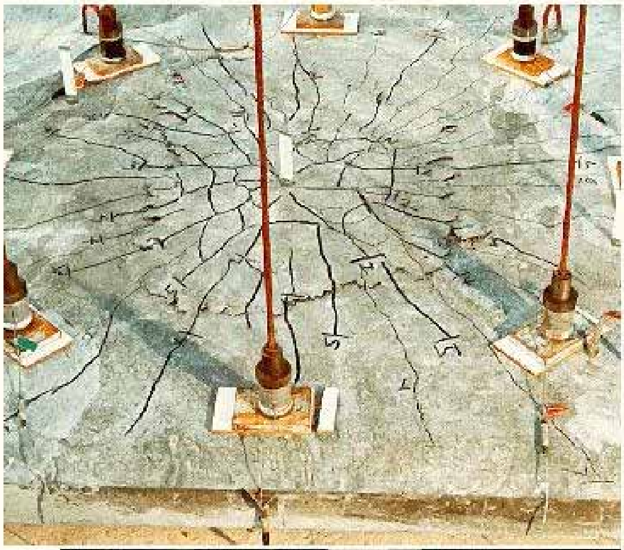
Rupture of concrete shell

Resistance

2 Shear Resistance

Tests show that cracks develop radially from the column position, culminating in a sudden and brittle failure on inclined faces of truncated cones or pyramids at an angle of about 35° to the horizontal.





Methods have been proposed for checking the stress on the failure planes, but the general method adopted is similar to that for transverse shear in beams, namely:

The shear force acting on a perimeter u around the loaded area is resisted by a nominal shear capacity (stress) τ_c acting over the average effective depth d of the section.

Shear resistance $V_{Rd1} = \tau_c u d$

Shear capacity

The shear capacity or average shear stress τ_c of a section is a mechanism for calculating the design shear resistance of a section without shear reinforcement, V_{Rd1} thus: (u is the length of the critical perimeter.)

$$V_{Rd1} = \tau_c u d$$

The empirical formula for the shear capacity, τ_c of a section is given by:

$$\tau_c = \tau_R k (1.2 + 40\rho_l)$$

τ_R is the basic design shear strength of concrete. Its *value* is taken as 25% of the 5% fractile value of the concrete tensile strength, thus: This is limited in the UK to the value for the concrete strength of $f_{ck} = 40$ N/mm².

$$\tau_R = 0.25 f_{ctk\,0.05} / \gamma_c$$

k is a coefficient to allow for the 'scale effect'. (d is in metres).

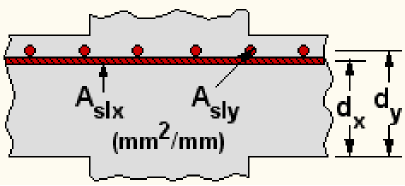
$$k = (1.6 - d) \nless 1$$

$1.2 + 40\rho_l$ allows for the combined effect of the concrete (1.2) and the tension steel ($40\rho_l$), (see Part 1 'Shear' for the derivation).

$$\rho_l = (\rho_{lx}\rho_{ly})^{0.5} \ngtr 0.015$$

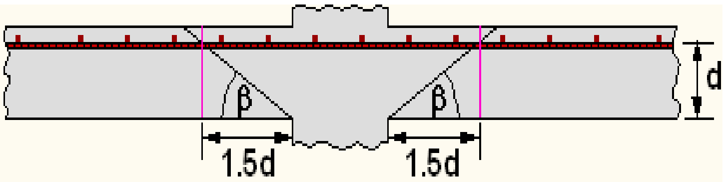
$$\rho_{lx} = A_{slx} / d_x$$

$$\rho_{ly} = A_{sly} / d_y$$



ρ_{lx}, ρ_{ly} are the reinforcement ratios in the x and y directions, neither of which should be less than 0.005. They are given by:

Critical perimeter

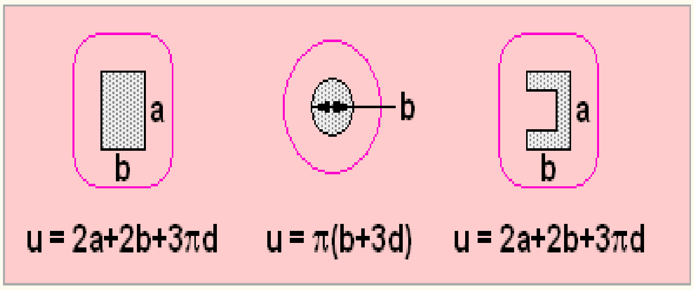


Comparisons with a large number of test results show that the closer the critical perimeter is to the loaded area, the greater is the influence of the size of the loaded area relative to the slab depth on the shear resistance of the slab.

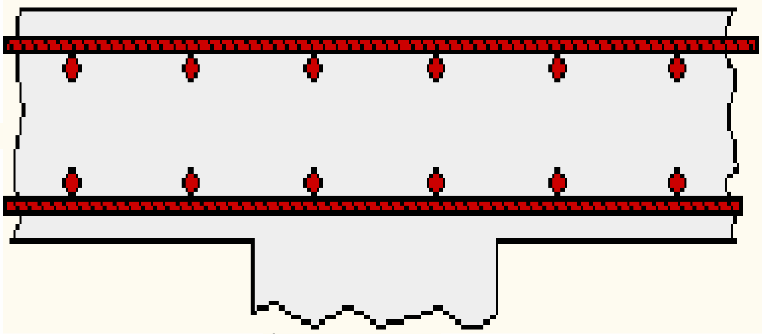
To be largely independent of this ratio, the distance of the critical perimeter from the face of the loaded area is taken as 1.5 d.

This gives an angle $\beta = 33.7^{\circ}$ (arctan(2/3)) with the horizontal, similar to the plane of failure.

These are the critical perimeters for some loaded areas. u is the length. Other shapes should be treated in a similar manner.



An example of critical perimeter



Internal column **400** mm square

h = 245 mm ; **cover = 25** mm

f_{ck}

d_x = 245 - 25 - 10 = 210 mm

d_y = 245 - 25 - 20 -10 = 190 mm

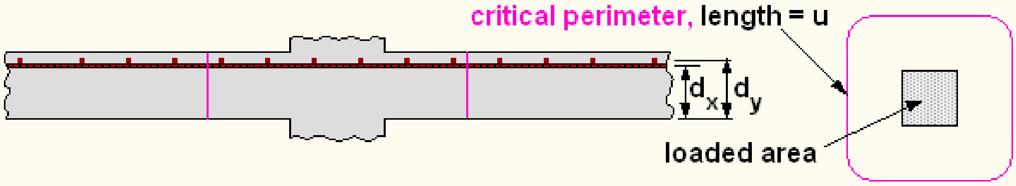
Therefore **d** = (210 + 190)/2 = **200** mm

Critical perimeter = 1.5 **d** = **300** mm

Therefore length of critical perimeter, **u** is :

4 x 400 + π x 3 x 200 = 3485 mm

Design procedure



- Calculate the total *effective* shear force, $V_{Sd,eff}$ at the face of the loaded area.
- Calculate the maximum shear resistance of the slab, V_{Rd2}. This is based on a maximum shear stress of $0.9 f_{ck}^{0.5}$ at the face of the loaded area, where u_l is the perimeter of the loaded area, and $d = (d_x + d_y)/2$.
- If $V_{Sd,eff} > V_{Rd2}$ either $V_{Sd,eff}$ must be reduced or V_{Rd2} increased.
- Calculate the shear capacity, τ_c at the critical perimeter.
- Calculate the design shear resistance of the slab, V_{Rd1} at the critical perimeter.
- If $V_{Sd,eff} > V_{Rd1}$ several design options are available to increase the shear resistance of the slab.

$$V_{Sd,eff} = \beta V_{Sd}$$

$$V_{Rd2} = 0.9 f_{ck}^{0.5} u_l d$$

$$\tau_c = \tau_R k (1.2 + 40\rho_l)$$

$$V_{Rd1} = \tau_c u d$$

Shear capacity - an example (continue)

Concrete C 35, $f_{ctk\ 0,05} = 2,2$ MPa

$$V_{Rd2} = 0.9 f_{ck}^{0.5} u_l d$$

$$V_{Rd2} = 0,9 \times 2,2^{0,5} \times 4 \times 0,4 = 2,14 \text{ MN}$$

$$V_{Rd1} = \tau_c u d$$

$$V_{Rd1} = 0,83 \times 0,2 \times 3,485 = 0,58 \text{ MN}$$

$$\tau_c = \tau_R k (1.2 + 40\rho_l)$$

$$\tau_c = 0,37 \times 1,4 \times (1,2 + 0,4) = 0,83 \text{ MPa}$$

$$\tau_R = 0.25 f_{ctk\ 0.05} / \gamma_c$$

$$\tau_R = 0,25 \times 2,2/1,5 = 0,37 \text{MPa}$$

$$k = (1.6 - d) \nless 1$$

$$k = 1,6 - 0,2 = 1,4$$

$$\rho_l = (\rho_{lx} \rho_{ly})^{0.5} \ngtr 0.015$$

$$\rho_{lx} = A_{slx} / d_x$$

$$\rho_{ly} = A_{sly} / d_y$$

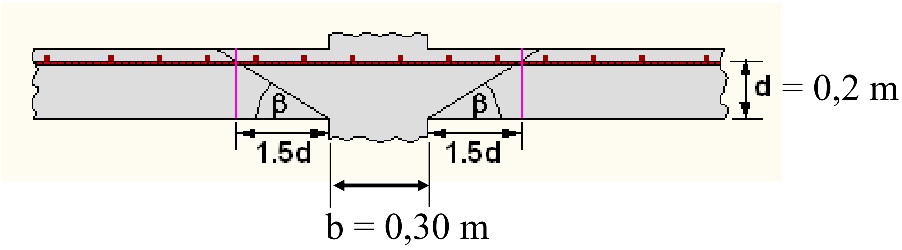
$$\rho \sim 0,01$$

Shear capacity depends on:
- concrete grade
- perimeter
- effective depth
- reinforcement ratio

An example of circular column

$$V_{Rd1} = \tau_c u d$$

Concrete C40/50: f_{ctk} = 2,5 MPa, $\tau_c \approx$ 2,1 MPa – separate calculation



Circular column u = π(b+3d) = 2,8 m

V_{rd1} = 2,1 × 2,8 × 0,2 = 1,18 MN

Exam questions

- Failure modes of reinforced concrete beam
- What is shear stress
- Draw lines of principle stresses in a beam
- Explain truss analogy
- Types of shear reinforcements
- Fundamental shear theory
- Shear capacity of a beam without shear reinforcement
- Examples of punching shear
- Shear capacity of a slab without shear reinforcement
- An example of punching shear verification

Basic formulae

Component	Design resistance	Notes
Bending, simple reinforcement	$M_{Rd} = A_s f_{yd}\left(d - \dfrac{A_s f_{yd}}{2b f_{cd}}\right)$	d effective depth
Bending, approximation	$M_{Rd} \cong z A_s f_{yd}$	$z \cong 0{,}9\ d$ internal arm
Short column, no excentricity	$N_{Rd} = 0{,}8\ A_c f_{cd} + A_s f_{yd}$	0,8 a reduction factor
Shear, no additional shear reinforcement	$V_{Rd} = \tau_c\ b\ d$	$\tau_c \cong \tau_{Rd}\ k\ (1 + 40\ \rho)$ $k \cong 1{,}6 - d$
Punching shear, no additional shear reinf.	$V_{Rd} = \tau_c\ u\ d$	u length of critical perimeter